Power Rule: $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Constant Multiple Rule

$$\frac{d}{dx}\left[cf(x)\right] = cf'(x)$$

Sum/difference Rule

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

proof of difference rule

$$\frac{d}{dx}\left[f(x) - g(x)\right] = \lim_{h \to 0} \frac{\left(f(x+h) - g(x+h)\right) - \left(f(x) - g(x)\right)}{h}$$

$$= \lim_{h \to 0} \frac{\left(f(x+h) - f(x)\right) - \left(g(x+h) - g(x)\right)}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h} \right)$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} - \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x) - g'(x) \quad //$$

$f(x) = b^x \longrightarrow f'(x) = ?$ $\left(\begin{matrix} b>0 \\ b\neq 1\end{matrix}\right)$

$f(0) = b^0 = 1$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{b^h - 1}{h} = \ln b$$

If $b=2$, then $f'(0) < 1$;
if $b=3$, then $f'(0) > 1$.

Then it's reasonable there is a choice of b between 2 & 3
so that $f'(0)=1$.

The number we can choose for b to make $f'(0)=1$ is e $(\approx 2.71828\ldots)$

So $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$.

So if $f(x) = e^x$ $\quad (b^{x+y} = b^x b^y)$

$$\frac{d}{dx}[e^x] = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$$

$$\boxed{\frac{d}{dx}[e^x] = e^x}$$

Ex: ① $f(t) = 2t^3 - 3t^2 - 4t$

$$f'(t) = 2(3t^2) - 3(2t) - 4(1)$$

$$= 6t^2 - 6t - 4$$

② $f(x) = \sqrt{x} - \frac{1}{7}x^7 + 4e^x$

$$= x^{1/2} - \frac{1}{7}x^7 + 4e^x$$

$$\boxed{f'(x) = \frac{1}{2}x^{-1/2} - x^6 + 4e^x}$$

③ $g(x) = 2e^x - \frac{1}{\sqrt[4]{x^{11}}}$ $\qquad (x^{11})^{-\frac{1}{4}} = (x^{1/4})^{-11}$

$= 2e^x - x^{-11/4}$

$$\boxed{g'(x) = 2e^x + \frac{11}{4}x^{-15/4}}$$

Q: If $f(x) = xe^x$, what is $f'(x)$?

Product Rule: If f & g are differentiable, then

$$\boxed{\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)}$$

Ex: ① $f(x) = xe^x$

$f'(x) = \frac{d}{dx}[x] \cdot e^x + x \cdot \frac{d}{dx}[e^x]$

$= 1 \cdot e^x + x \cdot e^x$

$$\boxed{f'(x) = e^x + xe^x}$$

② $g(x) = x^2\sqrt{x} - \dfrac{e^x}{x}$

$$= x^{5/2} - e^x x^{-1}$$

$$g'(x) = \frac{5}{2}x^{3/2} - \left(e^x x^{-1} + e^x(-x^{-2})\right)$$

$$= \frac{5}{2}x^{3/2} - e^x x^{-1} + e^x x^{-2}$$

③ $f(x) = (x^2+4x-1)e^x$

$$f'(x) = (2x+4)e^x + (x^2+4x-1)e^x$$

Fun Facts

①

$$\frac{d}{dx}\left[f(x)g(x)h(x)\right] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

②

$$\frac{d^2}{dx^2}\left[f(x)g(x)\right] = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x)$$